Math 250 – Sect.3.1, 3.2: Extrema, Rolle's Theorem, Mean Value Theorem

I. EXTREMA

Definition of EXTREMA: *A picture is worth a thousand words!!*

Extreme Value Theorem: If f is continuous on a closed interval [a, b], then f has both a minimum and a maximum on that interval.

NOTE: "Extreme Values" refers to the FUNCTION values. WHERE they occur refers to the *x* values (or the ordered pairs).

Where do extreme values occur?

1. Relative (or local) Extrema MAY occur when the derivative is = 0 or undefined. **The *x* values where the derivative is = 0 or undefined are called the *critical values*.** Endpoints are never considered "relative" extrema.

2. Absolute Extrema MAY occur at the endpoints of a closed interval OR at critical values.

To find where a function has extreme values:

1. Find the *critical numbers* for the function.
2. Find the function values at both the critical numbers AND the endpoints of the interval to determine absolute extrema.

-example- Find the extrema of $f(x) = 3x^4 - 8x^3$ on the interval [-2, 3].

-example- Find the extrema of $f(x) = 4x - 6x^{2/3}$ on the interval [-1, 8]

II. The Mean Value Theorem and Rolle's Theorem

Again, *a picture is worth a thousand words!*

Mean Value Theorem: If f is continuous on the closed interval [a, b], and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that:

Rolle's Theorem: Same hypothesis as MVT, but additionally we know that f(a) = f(b). Thus, there exists a number *c* in (a, b) such that:

Picture:

-example- Consider $f(x) = x^2 - 5x + 4$, on the interval [-1, 6]. Show that Rolle's Theorem can be applied to *f* on this interval. Then, find the value(s) of *c* guaranteed by Rolle's Theorem.

-example- Find the value(s) of *c* guaranteed by the Mean Value Theorem for the function $f(x) = \sqrt{2-x}$ on the interval [-7, 2].

-example- A company introduces a new product for which the number of units sold S is

$S(t) = 200\left(5 - \frac{9}{2+t}\right)$ where t is the time in months.

a. Find the **average rate of change** of S(t) during the first year.

b. During what month of the first year does $S'(t)$ equal the average rate of change?

c. Sketch a graph of this function, identifying the information found in parts (a) and (b).